Maximum-Information Guidance for Homing Missiles

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A recently-defined information index is used to enhance the information content of minimum-control-effort trajectories for the homing missile intercept problem. Optimal planar intercept trajectories are obtained for a performance index which is control effort weighted by position information content. The missile and target are assumed to be operating at constant speed. The shooting method is used to compute the optimal paths; but because of the simplicity of the model, on-line optimization yielding a guidance law with information enhancement should be possible.

Nomenclature

A	$=\cos\phi/v_R$
a	= missile normal acceleration (ft/s^2)
B	$=\sin\phi/v_R$
c	= constant in measurement variance model (ft^{-2})
G	= augmented end-point function
H	= variational Hamiltonian
R	= range (ft)
t	= time (s)
v_R	= ratio of missile velocity to target velocity
V	= velocity
W	= weight
X, Y	= planar coordinates (ft)
α	= nondimensional missile normal acceleration
θ	= missile velocity angle
λ	= time-dependent Lagrange multiplier
ν	= constant Lagrange multiplier
ξ,η	= nondimensional relative coordinates
ρ	= nondimensional range
τ	= nondimensional time
ϕ	= missile velocity angle

Superscripts

$(\dot{\ })$	= derivative with respect to t
$(\ )'$	= derivative with respect to τ

Subscripts

f	= final point
M	= missile
R	= relative
T	= target
0	= initial point

Introduction

IN Ref. 1, the problem of enhancing the information content of angle measurements in a homing missile engagement is considered. While the dynamics used in the filter development are linear in the states (relative position, relative velocity, and target acceleration), the measurements are nonlinear in a rectangular coordinate frame. Hence, the trajectory followed by the missile affects the measurement sequence and, in turn, the ability of the filter to extract the states from the measurements. A scalar performance index representing a measure of the information content of the missile path is developed, and a maximum-information intercept trajectory is determined. Next, measurements are created along the maximum-information path and processed with an extended Kalman filter. It is shown that the filter performs considerably better for measurements made along the maximum-information path than it does for measurements made along a proportional-navigation path. In fact, the filter diverges from the true state along the proportional-navigation path and converges to the true state along the maximum-information path.

Since the trajectory determined from the scalar information performance index reported in Ref. 1 induces greatly improved state estimation results, its use in the development of an information-enhancement guidance law is investigated. Because of the complexity of the problem, the simplest-possible intercept problem is formulated, that is, two-dimensional motion of a constant velocity missile and target. The performance index is taken to be the control effort weighted by the information index, and solutions are obtained by the shooting method. However, to obtain initial values of the Lagrange multipliers required by the shooting method, the problem of minimizing just the control effort must be considered first. Then, by solving the weighted problem in stages (gradually increasing the weight from zero), the desired optimal trajectories can be obtained.

Statement of the Problem

The classical guidance law known as proportional navigation is a perturbation guidance law about a nominal intercept triangle. The intercept triangle is essentially a minimum-control-effort trajectory in the plane (see Fig. 1 for nomenclature). For a constant velocity, steerable missile and a constant velocity target moving in a straight line, the optimal control problem is stated as follows:

Find the missile normal-acceleration history $a(t)$ which minimizes

$$J=\frac{1}{2}\int_{t_0}^{t_f} a^2 \mathrm{d}t \tag{1}$$

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subject to dynamical constraints

$$\dot{X}_R = V_T\cos\phi - V_M\cos\theta$$
$$\dot{Y}_R = V_T\sin\phi - V_M\sin\theta$$
$$\dot{\theta} = a/V_M \tag{2}$$

and the prescribed boundary conditions

$$t_0 = 0, \quad X_{R_0} = R_0, \quad Y_{R_0} = 0, \quad \theta_0 \equiv \text{free}$$
$$t_f \equiv \text{free}, \quad X_{R_f} = 0, \quad Y_{R_f} = 0, \quad \theta_f \equiv \text{free} \tag{3}$$

This optimal control problem[2] admits the solution $a = 0$ or $\theta = \text{const}$. However, along this path, the filter is not able to estimate all of the states because the range along the line-of-sight is unobservable. To enhance state estimation, it is possible to weight the final time with a term associated with information content. The simplest form of this term is obtained by considering only the position-information part of the performance index developed in Ref. 1. With this term included, the performance index, Eq. (1), can be rewritten as

$$J = \frac{1-W}{2}\int_{t_0}^{t_f} a^2 \mathrm{d}t - W\int_{t_0}^{t_f} \frac{\mathrm{d}t}{1 + c(X_R^2 + Y_R^2)} \tag{4}$$

where W is the weight and c is a constant associated with the measurement variance model used in the filter. If $W = 0$, J is the control effort; and if $W = 1$, it becomes the information integral. Since a minimum is being sought and since the information is to be maximized, the minus sign is introduced to convert the maximization problem to a minimization problem. Finally, when actually implemented, it is envisioned that W would be related to the state estimation error covariance, increasing as the covariance increases.

At this point, the following nondimensional variables are introduced:

$$\xi = \sqrt{c}X_R, \quad \eta = \sqrt{c}Y_R, \quad \tau = \sqrt{c}V_M t,$$
$$v_R = V_M/V_T, \quad \alpha = a/cV_M^2, \quad \rho = \sqrt{c}R \tag{5}$$

In terms of these variables, the optimal control problem is to find the missile normal-acceleration history $\alpha(\tau)$ which minimizes the performance index

$$J = \frac{1-W}{2}\int_{\tau_0}^{\tau_f} \alpha^2 \mathrm{d}\tau - W\int_{\tau_0}^{\tau_f} \frac{\mathrm{d}\tau}{1 + \xi^2 + \eta^2} \tag{6}$$

subject to the system dynamics

$$\xi' = \cos\phi/v_R - \cos\theta,$$
$$\eta' = \sin\phi/v_R - \sin\theta,$$
$$\theta' = \alpha \tag{7}$$

and the prescribed boundary conditions

$$\tau_0 = 0, \quad \xi_0 = \rho_0, \quad \eta_0 = 0, \quad \theta_0 \equiv \text{free} \tag{8a}$$
$$\tau_f \equiv \text{free}, \quad \xi_f = 0, \quad \eta_f = 0, \quad \theta_f \equiv \text{free} \tag{8b}$$

This optimal control problem does not yield an analytical solution and is solved with the numerical optimization method known as the shooting method. Because of the sensitivity of the shooting method to initial guesses, the problem is solved analytically for $W = 0$ to obtain Lagrange multipliers. Then, with these multipliers as initial guesses, the shooting method is converged for a small value of W. W is increased, and the process is repeated with the last converged multipliers as initial guesses.

Minimum Control-Effort Problem

For the case where $W = 0$, the variational Hamiltonian and the augmented end-point function are given by

$$H = \alpha^2/2 + \lambda_1(A - \cos\theta) + \lambda_2(B - \sin\theta) + \lambda_3\alpha$$
$$G = \nu_1\xi_f + \nu_2\eta_f \tag{9}$$

where $\lambda_i (i = 1,2,3)$ is a time-varying Lagrange multiplier, $\nu_i (i = 1,2)$ is a constant Lagrange multiplier, $A = \cos\phi/v_R$, and $B = \sin\phi/v_R$. The Euler-Lagrange equations[2] for λ lead to

$$\lambda_1' = -H_\xi = 0 \tag{10a}$$
$$\lambda_2' = -H_\eta = 0 \tag{10b}$$
$$\lambda_3' = -H_\theta = -\lambda_1\sin\theta + \lambda_2\cos\theta \tag{10c}$$

where the optimal control satisfies the optimality condition

$$H_\alpha = \alpha + \lambda_3 = 0 \tag{11}$$

Finally, the natural boundary conditions are

$$\lambda_{1_f} = G_{\xi_f} = \nu_1$$
$$\lambda_{2_f} = G_{\eta_f} = \nu_2$$
$$\lambda_{3_f} = G_{\theta_f} = 0, \quad \lambda_{3_0} = G_{\theta_0} = 0$$
$$H_f = \alpha_f^2/2 + \lambda_{1_f}(A - \cos\theta_f) + \lambda_{2_f}(B - \sin\theta_f) + \lambda_{3_f}\alpha_f = 0 \tag{12}$$

It is observed that the absolute minimum control effort is achieved when $\alpha = 0$. Whether or not this can be the solution is now investigated. Equations (10a) and (10b) indicate that λ_1 and λ_2 are constants so that Eq. (10c) gives $\theta = \text{const}$. Hence, the system equations, Eq. (7), can be integrated subject to the final conditions of Eq. (8b) to obtain

$$0 = (A - \cos\theta)\tau_f + \xi_0, \qquad 0 = B - \sin\theta \tag{13}$$

which determines θ and τ_f as follows:

$$\sin\theta = B, \qquad \tau_f = \xi_0/(\cos\theta - A) \tag{14}$$

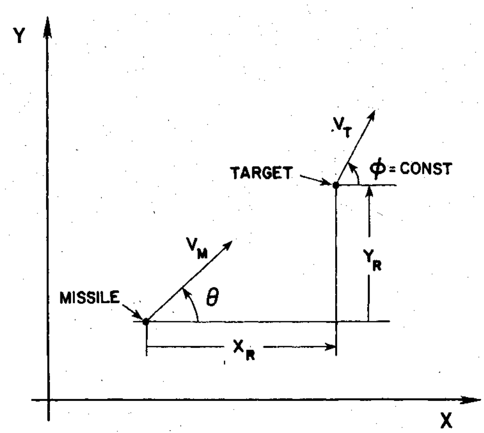

Fig. 1 Two-dimensional intercept geometry.

Next, Eq. (11) gives $\lambda_3 = 0$ which satisfies the natural boundary conditions of Eq. (12). Finally, Eqs. (10) and (12) lead to

$$\lambda_1 = 0, \qquad \lambda_2 = 0 \tag{15}$$

These values of λ will be used to begin the solution of the information-weighted minimum control-effort problem.

Minimum Information-Weighted Control-Effort Problem

For $W \neq 0$, the variational Hamiltonian and the augmented endpoint functions are defined as

$$H = \frac{1-W}{2}\alpha^2 - \frac{W}{1+\xi^2+\eta^2} + \lambda_1(A-\cos\theta) + \lambda_2(B-\sin\theta) + \lambda_3\alpha$$

$$G = \nu_1\xi_f + \nu_2\eta_f$$

Next, the differential equations for the λ's are given by

$$\lambda_1' = -2W\xi/(1+\xi^2+\eta^2)^2$$
$$\lambda_2' = -2W\eta/(1+\xi^2+\eta^2)^2$$
$$\lambda_3' = -\lambda_1\sin\theta + \lambda_2\cos\theta \tag{17}$$

while the optimal control must satisfy

$$(1-W)\alpha + \lambda_3 = 0 \tag{18}$$

Finally, the natural boundary conditions lead to

$$\lambda_{1_f} = \nu_1, \quad \lambda_{2_f} = \nu_2, \quad \lambda_{3_f} = 0, \quad \lambda_{3_0} = 0$$

$$-\frac{W}{1+\xi_f^2+\eta_f^2} + \lambda_{1_f}(A-\cos\theta_f) + \lambda_{2_f}(B-\sin\theta) = 0 \tag{19}$$

Unfortunately, this optimal control problem does not yield an analytical result so that numerical methods must be employed. Here, the shooting method[4] is used to solve the corresponding two-point boundary-value problem (TPBVP). It is formed by solving Eq. (18) for the control and eliminating α from the remaining equations to obtain the differential system

$$\xi' = A - \cos\theta$$
$$\eta' = B - \sin\theta$$
$$\theta' = -\lambda_3/(1-W)$$
$$\lambda_1' = -2W\xi/(1+\xi^2+\eta^2)^2$$
$$\lambda_2' = -2W\eta/(1+\xi^2+\eta^2)^2$$
$$\lambda_3' = -\lambda_1\cos\theta + \lambda_2\sin\theta \tag{20}$$

and the boundary conditions

$$\tau_0 = 0, \quad \xi_0 = \rho_0, \quad \eta_0 = 0, \quad \lambda_{3_0} = 0$$
$$\xi_f = 0, \quad \eta_f = 0, \quad \lambda_{3_f} = 0$$
$$-\frac{W}{1+\xi_f^2+\eta_f^2} + \lambda_{1_f}(A-\cos\theta_f) + \lambda_{2_f}(B-\sin\theta_f) = 0 \tag{21}$$

The TPBVP is solved by using the initial Lagrange multipliers for $W=0$ and a small value of W. Then, as W is increased, the initial guess for the λ's is the converged set for the

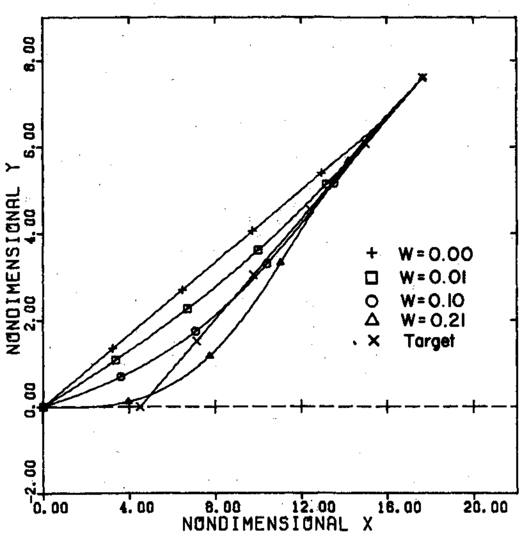

Fig. 2 Information-enhanced optimal intercept paths.

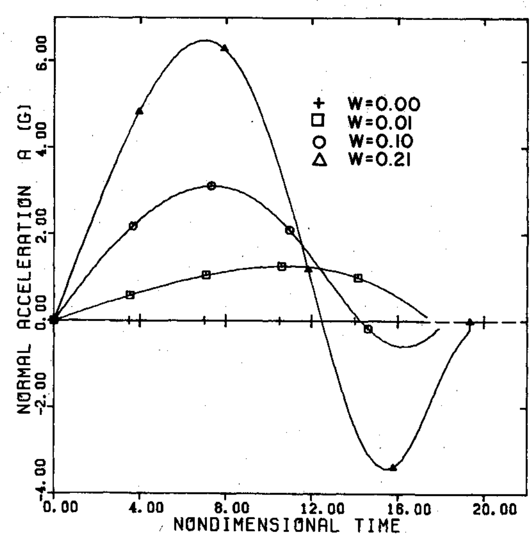

Fig. 3 Normal acceleration histories.

Table 1 Summary of numerical results[a]

W	θ_0, deg	λ_{1_0}	λ_{2_0}	Information content	Control effort	t_f, s
0.00	22.6199	0.00	0.00	5.2632	0.0000	9.2679
0.01	17.5956	−.00259	−0.00330	5.6373	.0025	9.3577
0.10	9.8100	−.02309	−.01244	5.7782	.0102	9.6648
0.21	−.8789	−.04570	−.01342	5.8825	.0268	10.4456

[a] $\rho_0 = 4.5$ (3000 ft), $v_R = 1.3$, $\phi = 30$ deg, $\tau_f =$ free, $\theta_0 =$ free.

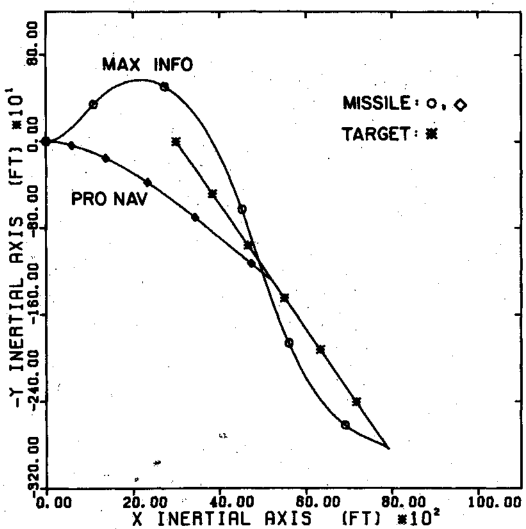

Fig. 4 Maximum information path, horizontal plane.

previous W. This procedure leads to the results presented in Table 1 and Figs. 2 and 3 for the case where the initial range is 3000 ft, the velocity ratio is $v_R = V_M/V_T = 1.3$, and the target direction is $\phi = 30$ deg.

It is noted from Table 1 that as W increases the control effort, the information content and the final time increase. The corresponding trajectories are shown in Fig. 2. For increasing W, the trajectories tend to move more toward a tail chase and oscillate back and forth behind the target. Also, the normal acceleration required to perform the maneuver, presented in Fig. 3, increases with W. For $W = 0.21$, the highest normal acceleration required is approximately 6 g.

The trajectories of Fig. 2 are similar to those obtained in Ref. 1 where the performance index is just information. For comparison purposes, the horizontal projection of the maximum information path of Ref. 1 is illustrated in Fig. 4. Note the similarity with $W = .21$ of Fig. 2.

Finally, solutions have only been obtained for values of W up to around 0.21. For $W > 0.21$, the shooting method is unable to converge to a solution. It is felt that the difficulty is caused by the minus sign in the performance index of Eq. (6). At some point, the missile can accumulate information faster than spending control to accomplish the intercept. Hence, the missile can wander around, accomplish the intercept at $t_f = \infty$, and generate $J = -\infty$.

Discussion and Conclusions

A recently-defined information index has been used to enhance the information content of minimum control-effort trajectories for the homing missile intercept problem. Optimal information-weighted trajectories have been obtained and display the desired characteristic, that is, maneuvering for the sake of increasing information content. Because of the simplicity of the model assumed here, it should be possible to compute these optimal trajectories on line and, hence, have a mechanizable guidance law for information enhancement.

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